

Quantum Field Theory of Dual Fock Space II

—The Orthogonal Relation of State Vectors—

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1. Introduction

In the last section of previous paper,¹ I wrote

$$” \langle t' \cdot |t \rangle \equiv \langle t'|t \rangle = \langle t|t \rangle' = 0. \quad (1)$$

This expression is verified by rewriting this in the matrix form.”

The purpose of this paper is to give the proof of this expression (1).

In section 2 the matrix representation of field operators is given. In the final section a speculation about the implication of expression (1) is given.

2. The orthogonal relation of state vectors

We define the extended ket vector $|v_k \rangle$ with its components v_k as a column vector. That is

$$|v_k \rangle = \begin{pmatrix} \vdots \\ v_{-2} \\ v_{-1} \\ \frac{v_{-0}}{v_{+0}} \\ v_1 \\ v_2 \\ \vdots \end{pmatrix},$$

where the line “—” inside the parenthesis is *the center line*.

There are two center components $v_{\mp 0}$ on both sides of the center line and the number of components of this ket vector is even, although it is infinite. The components of this vector extend both upward and downward limitlessly.

The corresponding bra vector $\langle v_k|$ is defined by

$$\langle v_k| = \left(\cdots \quad v_{-2} \quad v_{-1} \quad v_{-0} \quad | \quad v_{+0} \quad v_1 \quad v_2 \quad \cdots \right),$$

where the line “|” inside the parenthesis is also *the center line*.

Thus, for example, the scalar product of $|u_j \rangle$ and $|v_k \rangle$ is written as

$$\langle u_j|v_k \rangle = \sum_{k=-\infty}^{\infty} \delta_{jk} u_j v_k = \sum_{j=-\infty}^{\infty} u_j v_j = \langle v_k|u_j \rangle,$$

where we must take $j, k = \mp 0$ into account.

The state vectors and the field operators are defined for two cases. (We express these confining themselves for the case of one degree of freedom. The generalization to the arbitrary degrees of freedom is obvious.)

1) The operators of boson

The n -particle state of boson $|n\rangle, (|n'\rangle')$ in Fock space (in counter Fock space) is as follows.

$$|n\rangle = \begin{pmatrix} \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{pmatrix}, \quad |n'\rangle' = \begin{pmatrix} \vdots \\ 0 \\ 1 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \end{pmatrix}$$

, where

$$v_j = \begin{cases} 0 & (\text{if } j \leq -0) \\ \delta_{jn} & (\text{if } j \geq +0) \end{cases}, \quad v_{j'} = \begin{cases} \delta_{j',-n'} & (\text{if } j' \leq -0) \\ 0 & (\text{if } j' \geq +0) \end{cases}$$

respectively, and δ_{ab} is Kronecker's delta.

Then we define the number operator $N(N')$ in Fock space (in counter Fock space) by

$$N|n\rangle = a^\dagger a|n\rangle = n|n\rangle, \quad N'|n'\rangle' = a a^\dagger|n'\rangle' = n'|n'\rangle'. \quad (2)$$

That is,

$$a|n\rangle = \sqrt{n}|n-1\rangle, \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle, \quad (3)$$

and

$$a^\dagger|n'\rangle' = \sqrt{n'}|n'-1\rangle', \quad a|n'\rangle' = \sqrt{n'+1}|n'+1\rangle'. \quad (4)$$

Hence, $a|0\rangle = 0, a^\dagger|0\rangle' = 0$.

Thus we can get immediately

$$\langle n'|' \cdot |n\rangle \equiv \langle n'|'n\rangle = \langle n|n'\rangle' = 0. \quad (5)$$

From this expression it follows straightforwardly

$$\langle t|'t\rangle = \langle t|t\rangle' = 0. \quad (6)$$

Hence,

$$[b, b^\dagger]_+ \equiv bb^\dagger + b^\dagger b = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \equiv \left(\begin{array}{c|c} I & 0 \\ \hline 0 & I \end{array} \right) \equiv I,$$

and,

$$b^2 = 0, \quad b^{\dagger 2} = 0;$$

$$[b, b]_+ \equiv bb + bb = 0, \quad [b^\dagger, b^\dagger]_+ \equiv b^\dagger b^\dagger + b^\dagger b^\dagger = 0;$$

$$b|1 \rangle = |0 \rangle, \quad b|0 \rangle = 0, \quad b|0 \rangle' = |1 \rangle', \quad b|1 \rangle' = 0;$$

$$b^\dagger|1 \rangle' = |0 \rangle', \quad b^\dagger|0 \rangle' = 0, \quad b^\dagger|0 \rangle = |1 \rangle, \quad b^\dagger|1 \rangle = 0.$$

Thus the expression (6) was proved explicitly.

3. Discussion

We interpret about the implication of the expression (6) that it means that the positive(the negative) energy particle does not interacts with any negative(positive) energy particle through the electro-magnetic and any other kinds of interaction except for the gravitational interaction. Thus we may speculate that we can identify the negative energy which belongs to the counter Fock space to the dark energy.

Reference

1. M. Mizouchi, Bull. of Kurashiki Univ, of Science and the Arts **No.12**, (2007) 87.

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In this paper I would like to present and propose the following three items.

1. The extended matrix as the expression to represent field operators.
2. Fock space and counter Fock space are orthogonal to each other.
3. A speculation that the particles with the negative energy which belong to the counter Fock space are identified to the substances with the dark energy.