

Prototypes for Teaching Materials Based on the Golden Ratio

Caspar SCHWABE¹, Chris WALTON¹, Mamoru WATANABE²

College of the Arts¹, College of Science & Industrial Technology²

Kurashiki University of Science and the Arts

2640 Nishinoura, Tsurajima-cho, Kurashiki-shi, Okayama 712-8505, Japan

(Received October 1, 2011)

1. Introduction

This paper describes a set of four prototypes based on the golden ratio developed as teaching materials in design and mathematics lessons. All fabrication and construction used facilities at Kurashiki University of Science and the Arts, College of the Arts:

- i. The Golden Trisector designed by Caspar Schwabe
- ii. The GRF scale designed by Mamoru Watanabe
- iii. A new Tangram and the Pentangram designed by Mamoru Watanabe
- iv. A tile prototype by Chris Walton based on Penrose tiling

The golden ratio has aroused fascination since ancient times. Its properties are found in the Pyramids of Egypt and the Parthenon of Greece. In the modern age there are many articles with dimensions of the golden ratio—credit cards, digital cameras, cell phones, TV screens, computer displays, and others.

Plato stated the golden ratio as ([1]): Two quantities are in the golden ratio if the ratio of the sum of the quantities to the large quantity is equal to the ratio of the quantity to the smaller one. Expressed geometrically, the total length $a + b$ is to the length of the longer segment a as the length of a is to the length of the shorter segment b . We can express this algebraically as:

$$\frac{a+b}{a} = \frac{a}{b}$$

In this article this golden ratio is denoted by Φ . Since $1 + \frac{b}{a} = \frac{a}{b}$, $1 + \frac{1}{\Phi} = \Phi$, then we have

$$\Phi^2 = \Phi + 1.$$

This equation has one positive and irrational number

$$\Phi = \frac{1 + \sqrt{5}}{2} = 1.618033988...$$

If we denote $\phi = \Phi - 1$, then $\phi \cdot \Phi = \Phi^2 - \Phi = 1$ by multiplying Φ both sides, then

$$\phi = \frac{1}{\Phi} = 0.618033988...$$

2. The Golden Trisector

The traditional golden compass has two or three legs, or gnomons, and divides a given length according to the golden section into a so-called Major and minor section (Figure 1): $M = \Phi$, $m = 1$

In contrast, the Golden Trisector consists of four legs, or gnomons, and therefore divides a given length into three sections: Major, minor and $M-m$ (Major minus minor).

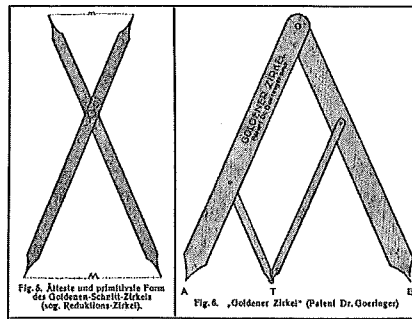


Figure 1: A traditional golden compass

A drawback of the traditional golden compass is that it does not show the sequence of the golden ratio, whereas the Golden Trisector easily identifies the golden succession with its three sections (Figure 2).

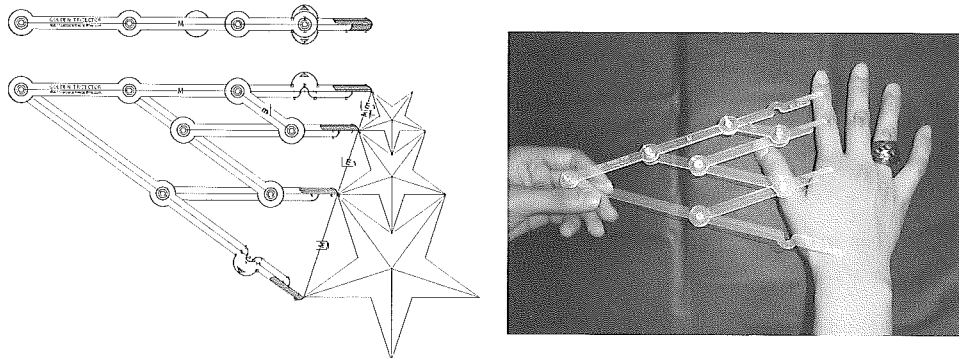


Figure 2: Drawing design for the Golden Trisector and the completed acrylic prototype

The design-innovative characteristic of the Golden Trisector is that it can be folded together into a thin, straight form similar to a pen. Its light weight also makes it practical to carry for the design student. Opened into a right-angled compass, the two viewfinders of the Golden Trisector frame a landscape into the Golden proportion. The Golden Trisector is the perfect instrument to reveal the innate natural beauty which exists around us.

The Golden Trisector was first drawn in DraftBoard CAD software, then cut from fluorescent, black light sensitive acrylic on a computer controlled laser cutting machine. Joints are secured by seven nylon plastic screws.

3. The GRF Scale

We produced a tool to determine if two adjacent edges of a polygon contain the golden ratio $1:\Phi$. While the traditional compass for this type of measurement is capable, this scale gives attention to the shape, not the length of a given polygon. The tool is named the Golden Ratio Figure Scale, or simply the GRF Scale.

(i) Drawing for the GRF Scale

The GRF scale consists of two parallelograms, both of which can swivel at corner joints (Figure 3). The edge lengths of the small parallelogram are in the golden ratio. The larger parallelogram has a track running the length of one edge. The small parallelogram is fastened at its diagonal corners onto this track. By manipulating diagonal corners, we can slide the small parallelogram along the track, and narrow or widen both parallelograms smoothly as our needs require. Drawings were made in DraftBoard CAD software and acrylic prototypes were cut using a laser cutting machine. Joints are secured by eight nylon plastic screws.

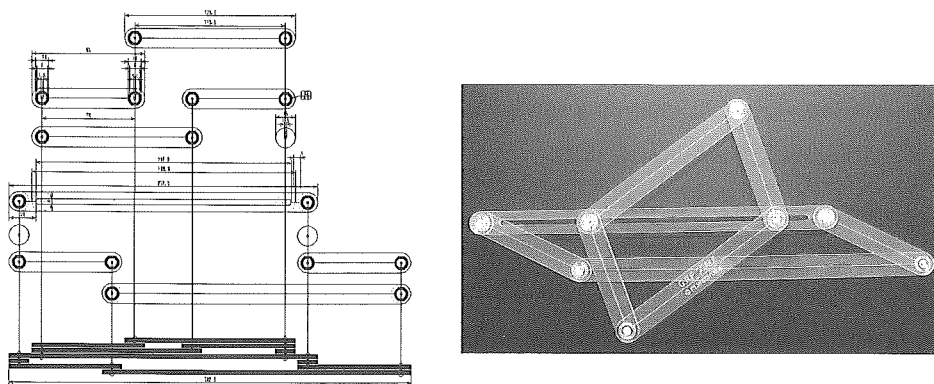


Figure 3: CAD drawing for the GRF Scale and the completed acrylic prototype

(ii) Using the GRF Scale

As an example, let us check whether adjacent edges (let them be e and f) of a credit card are in golden ratio, we can use the following procedure (Figure 4):

1. Position the card by aligning points S and A.
2. By sliding points D and B right or left, make the angle PAQ coincident to angle UST .
3. Move bar XY toward the position of segment UT using parallel movement. If segments PQ and UT are completely coincident, then edges e and f are in golden ratio. We can verify this by noting whether triangles PAQ and DAB are similar. If they are not similar, then edges e and f are not in golden ratio.

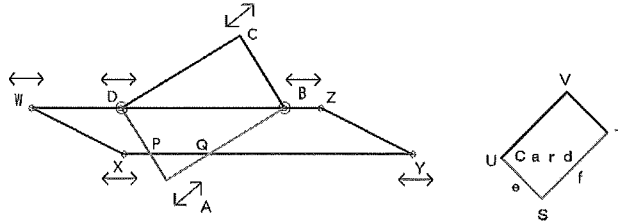


Figure 4: The moving parts of the GRF Scale, indicated by the arrows

4. Tangram and Pentangram

The tangram is a type of puzzle which dates to the 18th century. A formal definition of the tangram would be: Let F be a figure given by dividing a convex polygon into smaller sections and let G be a silhouette. Make silhouette G by arranging all pieces of F .

The history of the tangram is not always clear. Many people believe that it originated in China, but the oldest existent literature was published in Japan in 1742. An amazing variety of tangrams can be found in Rüdiger Thiele and Konrad Haase's fascinating book, *Teufelsspiele* ([2]).

(i) Drawing for the Pentangram

We used the golden ratio to produce a new type of tangram puzzle which is based on the division of a pentagon (Figure 5). We call it a Pentangram, from a combination of pentagon and tangram.

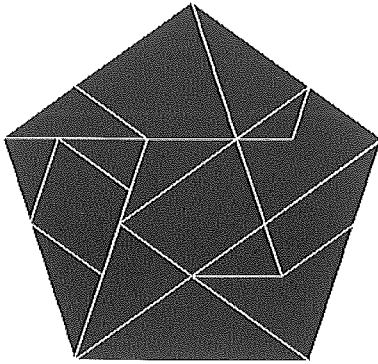


Figure 5: The Pentangram pieces

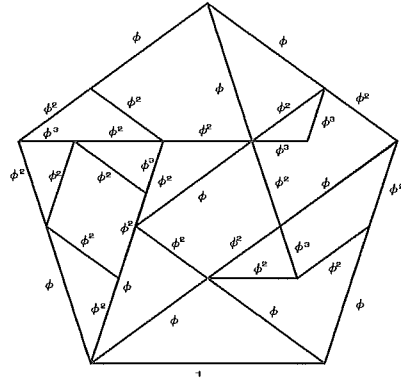


Figure 6: Lengths of their edges

(ii) Properties of the Pentangram and educational merit

1. The Pentangram is a totally new type of tangram consisting of 16 pieces made from the division of a pentagon.
2. Each piece except the rhombus has two adjacent edges in golden ratio. In the rhombus, the short edge and the long diagonal are also in golden ratio (Figure 5).
3. The corner angle of each piece is a multiple of 36° , so the system of angles is quite simple. A pentagon cannot create a 90° angle (Figure 5).
4. Lengths of all edges share four measurements that are in golden ratio (Figure 6).
5. Points 3 and 4 make aligning edges easier than for other tangrams.
6. To increase the variety of shapes, two concave pieces were included.
7. Numerous trials and tests reveal that making animal forms is easier with the Pentangram than for other tangrams.
8. Using all the pieces of one Pentangram set, we can make several silhouettes that are similar with the golden ratio. We can even make triple sets.
9. Fluorescent acrylic was used, which produces brightly illuminated and beautiful edges that are easy to see.
10. The Pentangram presents a challenging visual and mental exercise appropriate not only in a general school environment, but additionally beneficial as a rehabilitation tool for the handicapped. The size can be enlarged and the number of pieces reduced as necessary.

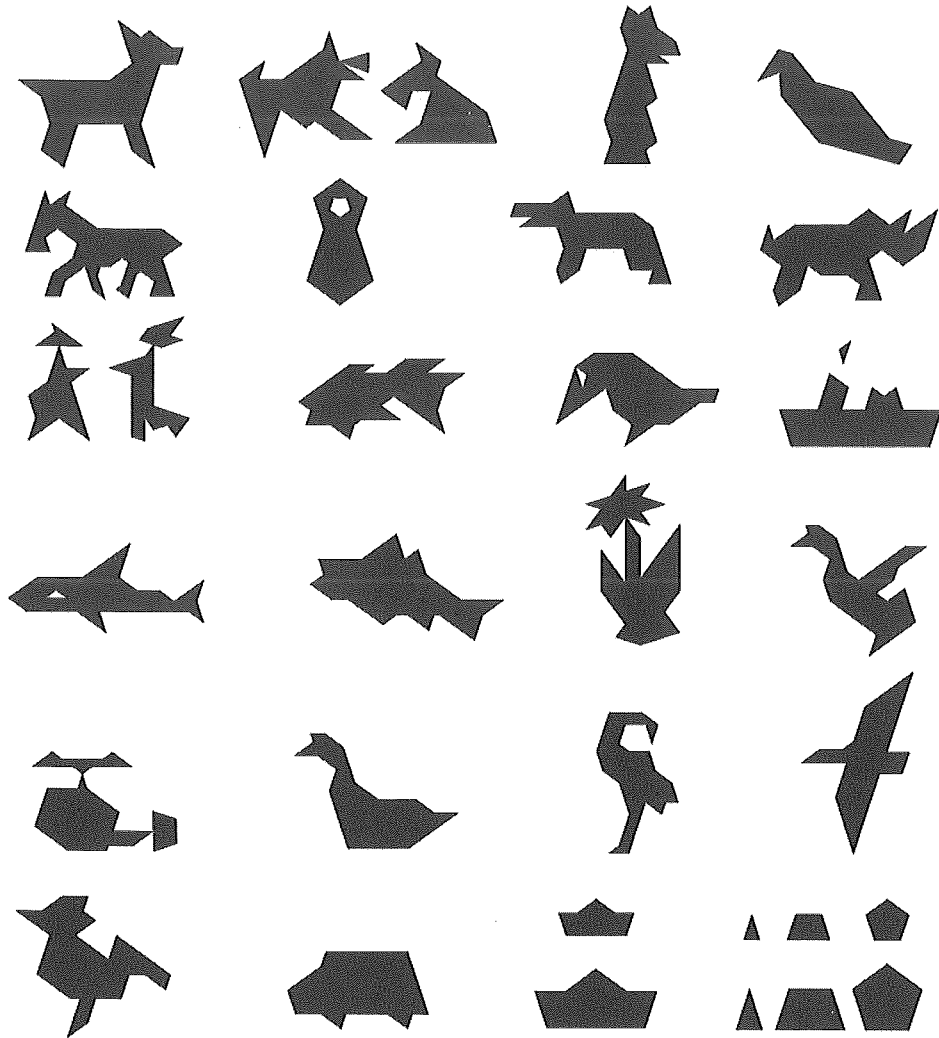


Figure 7: A sampling of silhouettes created with the Pentangram

5. A Tile Prototype based on Penrose tilings

For centuries, artists and architects have approximated the golden ratio in the dimensions of the golden rectangle, believing that the harmony of these proportions reveals a universal aesthetic ([3]). However, the golden ratio is present in a variation of forms, including the fantastic geometric surface tiling system known as Penrose tiling. The author adopted Penrose tiling for producing prototypes. The Penrose P3 tiling uses a fat rhombus (Figure 8, left) and a skinny (Figure 8, right) rhombus, or rhomb, which we can derive from a pentagon and a pentagram (Figure 8).

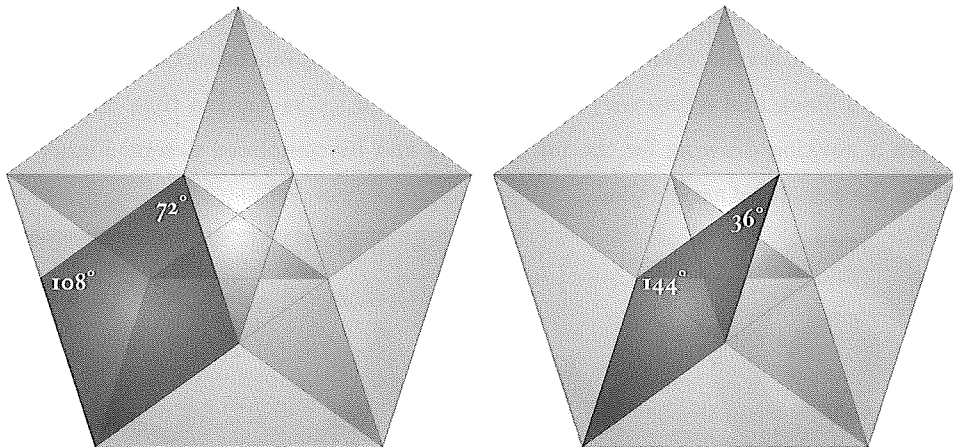


Figure 8: The two rhombuses, fat and skinny, used for this prototype

The English mathematician Sir Roger Penrose discovered in 1974 that with these two rhombuses, a tiling could be constructed on a two-dimensional plane that exhibits five-fold symmetry and is non-repeating—a finite number of parts producing something infinite and endlessly variable ([4]). Figure 9 demonstrates a small portion of one such example. The author's purpose was to draw designs on the tile surfaces, allowing for a variety of matching outcomes based on user preference. Figure 10 shows a small portion of these results.

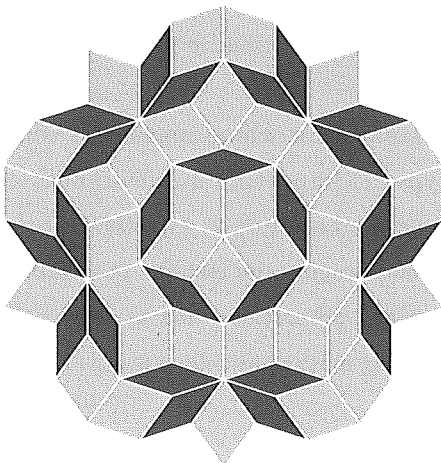


Figure 9: A Penrose tiling

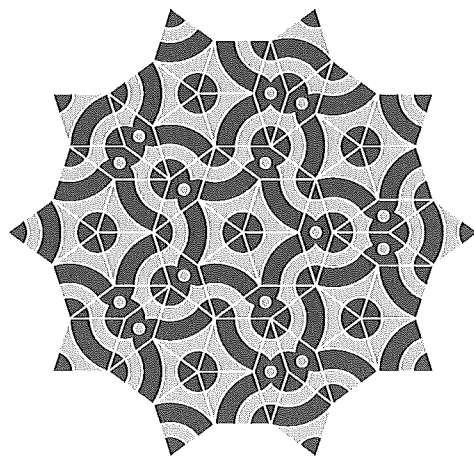


Figure 10: Tiling with surface design

(i) Drawing for Penrose Tiling Design

Tiles were drawn in DraftBoard CAD software, then imported into Adobe

Illustrator for the addition of surface designs using golden ratio dimensions. Mirrored silver and gold acrylic sheets were cut on a laser cutting machine. Cut pieces were bonded to black and white acrylic backings to produce reversible tiles. Figure 11 shows a working drawing for the fat rhombus and its completed design. Figure 12 shows the completed acrylic tiles.

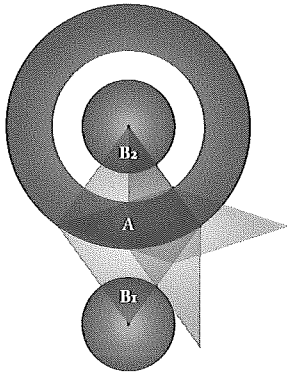


Figure 11: Drawing for the thick rhomb

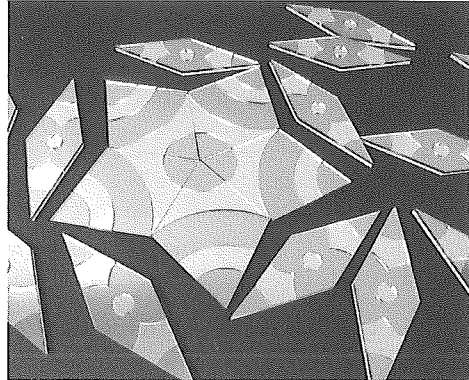


Fig 12: The completed fat and skinny rhombs

(ii) Educational Appeal

Penrose tiling presents a fascinating playfulness with its puzzle-like appearance, inspires creativity with a variety of unexpected design outcomes, and through hands-on experimentation teaches important lessons about mathematics in an enjoyable visual manner that appeals to many people.

For design students, four points of educational value were considered when researching and constructing this prototype: 1) imparting an awareness of complimentary relationships between design and mathematics, 2) investigating the unique visual properties of the golden ratio, 3) encouraging the creative possibilities of borrowing from natural forms that exhibit golden proportions, and 4) developing skills of construction and craftsmanship in expressing mathematically accurate design relationships.

6. Conclusion

This research was supported by a grant from Kurashiki University of Science and the Arts. The initial stage of this project occurred over a period of three months and focused on the development of prototypes which were exhibited at the Kake Museum of Art in Kurashiki from September 16 to October 23, 2011 (Figure 13). The exhibition

had an educational goal and was also opportunity to test the construction durability, the ease of handling and the ease of understanding the purpose of these teaching materials. The authors presented a public talk at the museum on September 17, 2011.

Research of this theme is ongoing to develop additional teaching materials and enhance classroom appeal. Future activity includes producing new types of tangrams based on the golden ratio, investigating a combination of tangrams and tilings, and a variety of new tiling designs based on Penrose tilings.



Fig 13: This first stage of research culminated in exhibition at the Kake Museum of Art

Acknowledgements

The authors would like to extend their sincere thanks and appreciation to Asata Matsumura, chief curator at the Kake Museum of Art, Kurashiki, and his staff for their assistance in preparing for the exhibition of these prototypes.

Bibliography

- [1] Scott Olson, *The Golden Section: Nature's Greatest Secret*, Walker Publishing Company, Inc., New York.
- [2] Rüdiger Thiele, Konrad Haase, *Teufelsspiele*, Uriania-Verlag, Berlin, 1991.
- [3] György Doczi, *The Power of Limits: Proportional Harmonies in Nature, Art, and Architecture*, Shambhala, Boston, 2005.
- [4] Tanioka Ichiro, *Escher and Penrose Tiles* (in Japanese), PHP Science World, 2009.

Abstract

Caspar SCHWABE¹, Chris WALTON¹, Mamoru WATANABE²

College of the Arts¹, College of Science & Industrial Technology²

Kurashiki University of Science and the Arts

2640 Nishinoura, Tsurajima-cho, Kurashiki-shi, Okayama 712-8505, Japan

(Received October 1, 2011)

During the spring term of 2011, the authors received a grant from Kurashiki University of Science and the Arts to conduct multi-disciplinary research investigating complimentary principals between visual design and mathematics, specifically the golden ratio. This paper reports on a set of four prototype designs developed as teaching materials for future classroom instruction:

- i. The Golden Trisector—a new golden compass that shows the sequence of the golden ratio and the golden succession with its three sections
- ii. The GRF scale—a tool for determining if two adjacent edges of a polygon are in golden ratio by measuring an object's shape, not the size of its edges.
- iii. The Pentangram—a new kind of tangram based on multiple parts derived from a pentagon.
- iv. A tile prototype which attempts to create matching surface designs on Penrose tiling.

We briefly describe our research motivation, the methods of fabricating and constructing prototypes using facilities within the College of the Arts, the outstanding characteristics of these designs and the educational merit of the resulting teaching materials.

The initial stage of this research resulted in an exhibition of prototypes at the Kake Musuem of Art in Kurashiki from September 16 to October 23, 2011.